Problem Sheet 4

Choose some of these statements to prove. Once you have an outline proof, have a go at writing out a clear and complete proof using appropriate mathematical notation, and then ask your mentor to check it.

1. $$x_1 = 2^2 + 3^2 + 6^2$$
   $$x_2 = 3^2 + 4^2 + 12^2$$
   $$x_3 = 4^2 + 5^2 + 20^2$$
   Show that x_n is always a perfect square.

2. Prove that if you add 1 to the product of four consecutive whole numbers the answer is ALWAYS a perfect square.

3. Take any two numbers between 0 and 1. Prove that the sum of the numbers is always less than one plus their product.

   That is, if $0 < x < 1$ and $0 < y < 1$ then prove

   $$x + y < 1 + xy$$

   .

4. $2(5^2 + 3^2) = 2(25 + 9) = 68 = 64 + 4 = 8^2 + 2^2$
   $2(7^2 + 4^2) = 2(49 + 16) = 130 = 121 + 9 = 11^2 + 3^2$
   Prove that if you double the sum of two squares you get the sum of two squares.

5. Prove that if the integer n is divisible by 4 then it can be written as the difference of two squares.

6. Prove that if $a^2 + b^2$ is a multiple of 3 then both a and b are multiples of 3.

7. Prove that there is no arithmetic progression containing all three of $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$.